Nonhomogeneous Navier-Stokes Equations and Control Problems

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Abstract: Let Ω be a bounded connected domain with Lipschitz boundary $\Gamma = \Gamma_D \cup \Gamma_C \cup \Gamma_N$, where Γ_C is regular. We are interested in the following nonhomogeneous boundary value problems for the Navier-Stokes equations:

$$\frac{\partial \mathbf{u}}{\partial t} - \nu \Delta \mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u} + \nabla p = 0, \quad \text{div } \mathbf{u} = 0 \quad \text{in } \Omega \times (0,T),$$

$$\mathbf{u} = 0 \ \text{ on } \Gamma_D \times (0,T), \quad \mathbf{u} = \mathbf{g} \ \text{ on } \Gamma_C \times (0,T),$$

$$\nu \frac{\partial \mathbf{u}}{\partial n} - p\mathbf{n} = 0 \ \text{ on } \Gamma_N \times (0,T), \quad \mathbf{u}(0) = \mathbf{u}_0 \text{ in } \Omega,$$

where $\mathbf{g}$ is nonhomogeneous boundary condition, $\mathbf{u}_0$ is the initial condition.

We need some geometrical conditions on the domain to study the regularity of solutions to the system. These results will allow us to go further and to study optimal control problems, when the control is applied along Γ_C.

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