

# Massively Parallel Geostatistical Inversion of Coupled Processes in Heterogeneous Porous Media

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**Abstract:** The quasi-linear geostatistical approach is an inversion scheme that can be used to estimate the spatial distribution of a heterogeneous hydraulic conductivity field. The estimated parameter field is considered to be a random variable that varies continuously in space, meets the measurements of dependent quantities (such as the hydraulic head, the concentration of a transported solute or its arrival time) and shows the required spatial correlation (described by certain variogram models). This is a method of conditioning a parameter field to observations. Upon discretization, this results in as many parameters as elements of the computational grid. For a full three dimensional representation of the heterogeneous subsurface it is hardly sufficient to work with resolutions (up to one million parameters) of the model domain that can be achieved on a serial computer.

The forward problems to be solved within the inversion procedure consists of the elliptic steady-state groundwater flow equation and the formally elliptic but nearly hyperbolic steady-state advection-dominated solute transport equation in a heterogeneous porous medium. Both equations are discretized by Finite Element Methods (FEM) using fully scalable domain decomposition techniques. Whereas standard conforming FEM is sufficient for the flow equation, for the advection dominated transport equation, which rises well known numerical difficulties at sharp fronts or boundary layers, we use the streamline diffusion approach. The arising linear systems are solved using efficient iterative solvers with an AMG (algebraic multigrid) pre-conditioner.

During each iteration step of the inversion scheme one needs to solve a multitude of forward and adjoint problems in order to calculate the sensitivities of each measurement and the related cross-covariance matrix of the unknown parameters and the observations. In order to reduce interprocess communications and to improve the scalability of the code on larger clusters another level of parallelization has been added.

An alternative numerical scheme (locally adaptive DG-FEM) which incorporates upwinding in a natural way for the solution of the convection dominated transport problem is being explored.

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