Sparsity Regularization of the Diffusion Coefficient Identification Problem: Theory and Numerical Solutions

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Abstract: In this paper, we investigate sparsity regularization for the diffusion coefficient identification problem: identify the coefficient $\sigma \in L^\infty(\Omega)$ in the equation

$$-\nabla.(\sigma\nabla\phi) = y \; in \; \Omega, \; \phi = 0 \; on \; \partial\Omega,$$

from noisy data $\phi^\delta \in H_0^1(\Omega)$ of ϕ with

$$\|\phi - \phi^\delta\|_{H^1(\Omega)} \leq \delta \; (\Omega \subset \mathbb{R}^d, d \geq 2).$$

In many applications, the parameter σ that need to recover has a sparse representation, i.e. the components of nonzero of $\sigma - \sigma^0$ are finite in an orthonormal basis (or frame) of $L^2(\Omega)$. The sparsity of $\sigma - \sigma^0$ promotes to use sparsity regularization.

Using the energy functional approach incorporating with sparsity regularization, regularized solutions of this problem are minimizers of the functional

$$\int_\Omega \sigma|\nabla F_D(\sigma)y - \phi^\delta|^2 dx + \alpha\Phi(\sigma - \sigma^0),$$

where $F_D(\cdot)y : D(F_D) \to H_0^1(\Omega), \sigma \mapsto \phi$ with ϕ, the solution of the above problem, $\alpha > 0$ is the regularization parameter; $\Phi(\vartheta) := \sum \omega_k |\langle \vartheta, \varphi_k \rangle|^p \; (1 \leq p \leq 2)$, $\{\varphi_k\}$ is an orthonormal basis (or frame) of $L^2(\Omega)$ and $\omega_k \geq \omega_{min} > 0$ for all k.

The purposes of the paper is to consider the well-posedness, convergence rates of the method and present fast algorithms for finding minimizers of the above functional. The efficiency of the algorithms and numerical solutions are illustrated by a particular example. The advantage of the energy functional approach is to deal with a convex problem. Therefore, the well-posedness of the method is proved without requiring the weakly closedness of F_D and some fast algorithms can be applied for such a convex minimization problem. Furthermore, convergence rates of the method is obtained under a simple source condition, which is not require the smallness or its generalizations as the least squares approach used.

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