# A Mathematical Study of Sprinting on Artificial Legs

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**Abstract:** The remarkable performance of the double amputee 400 m sprinter Oskar Pistorius initiated a big discussion that his running prostheses actually might give him an advantage over able-bodied sprinters. He uses carbon fiber Cheetah devices (Össur) that have spring-like properties; and the assumption was that the high passive torques and the lower moments of inertia of the prosthetical lower leg more than compensate for the absence of an active ankle torque. The purpose of our research is to use mathematical models and optimal control techniques and combine them with experimental data in order to better understand the underlying mechanics and control of sprinting on prosthesis and to bring new insights into the still ongoing discussion.

We established rigid multibody system models for the hybrid dynamics of able bodied as well as double amputee sprinters. In the present study, we use models in the sagittal plane with nine bodies and 11 degrees of freedom. In the able bodied case, there are torque actuators at all eight internal joints; in the double amputee case, the actuators at the ankles are replaced by linear spring damper elements, but the other six actuators remain. The modeled subject has an overall weight of 83.3 kg and a height of 1.85 m corresponding to the data of Oskar Pistorius. For the anthropometric geometry and inertia data, we use an extrapolation of the de Leva data to the desired height and weight, as well as data for the prostheses and the stump given by Brueggemann et al.

The description of running motions leads to multi-phase optimal control problems with free phase durations and state-dependent phase switching. We use different optimization criteria to generate sprinting motions and compare the results to experimental data. The choices of objective functions are based on physiological studies of the objectives and performance measures of 400 m sprinting. Periodicity and phase switching conditions as well as all physical bounds on state and control variables are considered. The phase order for running is fixed, but the phase times are left free in the optimization. Along with the control and state variable functions, we also optimized the spring and damper constants of the prostheses. The optimal control problem is solved using an efficient direct multiple shooting approach developed by Bock an co-workers.

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