Two-step Family of "Look-ahead" Linear Multistep Method for ODEs

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Abstract: We are concerned with numerical solutions of initial-value problem of ordinary differential equations (ODEs) given by

$$\frac{\mathrm{d}y}{\mathrm{d}x} = f(x, y) \quad (a \le x \le b), \quad y(a) = y_I,$$

where the unknown function y is a mapping $[a, b] \to \mathbb{R}^d$, the right -hand side function f is $[a, b] \times \mathbb{R}^d \to \mathbb{R}^d$ and the initial vector y_I is given in $\mathbb{R}^d$. Among many numerical solutions, we are interested in the discrete variable methods (DVMs) with a constant step-size h to generate the approximate solution y_n on the step-point $x_n = a + nh$. Recently we developed a new class of DVM, namely "look-ahead" linear multistep methods (LALMM).

The basic idea of LALMM is as follows: Assume that we look for the numerical solution of the $(n + k)$-th step-point when the back-values $y_n, y_{n+1}, \ldots, y_{n+k-1}$ and a preassigned *initial guess* $y^{[0]}_{n+k}$ are available. First, we look ahead for the $(n + k + 1)$-st step-point by

$$y^{[0]}_{n+k+1} + \alpha_k y^{[0]}_{n+k} + \sum_{i=0}^{k-1} \alpha_i y_{n+i} = h\left(\beta_k f(x_{n+k}, y^{[0]}_{n+k}) + \sum_{i=0}^{k-1} \beta_i f(x_{n+i}, y_{n+i})\right),$$

as a predictor. Then, we correct the look-for value by

$$y^{[1]}_{n+k} + \sum_{i=0}^{k-1} \alpha^*_i y_{n+i} = h\left(\beta^*_{k+1} f(x_{n+k+1}, y^{[0]}_{n+k+1}) + \beta^*_k f(x_{n+k}, y^{[0]}_{n+k}) + \sum_{i=0}^{k-1} \beta^*_i f(x_{n+i}, y_{n+i})\right)$$

until a local convergence is attained. A general theory of convergence order as well as linear stability criterion have been also developed.

In the talk, a two-step family of LALMM is explained with a special emphasis on its stability. The characteristic root of stability is of a quadratic polynomial. However, because of its feature of two-step family and "look-ahead" property, the dependence of the root to the variable $z = \lambda h$ is also quadratic. Therefore we take the Schur criterion for stability analysis. We present four fourth-order pairs of predictor and corrector and show that one of them has A-stability, while others $A(\theta)$-stability as well.

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