

Higher-order Exponential Rosenbrock-type Methods

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Abstract: For the time integration of large-scale systems of stiff differential equations,

$$u'(t) = F(t, u(t)), \quad u(t_0) = u_0$$

we consider the class of exponential Rosenbrock-type methods. These methods rely on the use of the matrix exponential and related functions of the full Jacobian of the problem. On the other hand, the methods are fully explicit and do not require the numerical solution of linear systems. We present a new approach to construct stiff order conditions. This allows us to derive new pairs of embedded methods of high order. As an example, we present a fifth-order method with five stages. The error analysis is performed in a semigroup framework of semilinear evolution equations in Banach spaces. We present convergence results for variable step size methods. To demonstrate the efficiency of the new integrators, we give some numerical experiments in MATLAB. In particular, we present numerical comparisons for semilinear parabolic PDEs in one and two space dimensions.

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