Solution of Inverse Problem of the Variational Calculus for Differential Equations of Second Order with Deviating Argument

G. A. Kurina[1]

Abstract: We consider the following equation with deviating argument

$$(A + \widetilde{B})y''(x) + Cy''(x - \theta) + \widetilde{D}y''(x + \theta) + E + \widetilde{F} = 0. \tag{1}$$

Here the coefficients A, B, C, D, E, F are functions that depend on x, $y(x)$, $y(x - \theta)$, $y'(x)$, $y'(x - \theta)$ and are twice continuously differentiable with respect to the arguments; $\widetilde{B} := B(x+\theta, y(x+\theta), y(x), y'(x+\theta), y'(x))$, where $B = B(x, y(x), y(x-\theta), y'(x), y'(x-\theta))$. Also, it is assumed that admissible functions $y(\cdot)$ satisfy the following conditions

$$y(x) = \varphi(x), \quad x \in [a - \theta, a]; \quad y(x) = \psi(x), \quad x \in [b - \theta, b]. \tag{2}$$

We study the inverse problem of the variational calculus for equation (1). Namely, we seek for an integral functional for which Euler's equation coincides with equation (1). We obtain conditions ensuring the solvability of the inverse problem for equation (1). Moreover, we show that the functional solving the inverse problem admits the following representation

$$J(y) = \int\limits_a^b \Big(K + \int\limits_0^{y'(x)} \Big(H - \int\limits_0^{y'(x-\theta)} C dy'(x - \theta)\Big) dy'(x)\Big) dx, \tag{3}$$

where the functions H and K are defined by means of coefficients in (1).

The results are demonstrated by illustrative examples.

Euler's equation for integral functionals with deviating argument and with conditions (2) can be found in, e.g., [1]. Under additional assumptions on coefficients in (1) and other conditions for admissible functions, necessary and sufficient conditions for the solvability of the inverse problem of the variational calculus have been obtained in [2].

References

[1] G. A. Kamenskii, *Funct. Differ. Equations*, **12** (2005), no.3-4, pp. 245–270.

[2] V. G. Zadorozhnii and G. A. Kurina, *Math. Notes*, **90** (2011), no.2, pp. 218–226.

[1] Department of Mathematics
Voronezh State Academy of Forestry Engineering
Timiryazeva, 8, 394087 Voronezh, Russia
kurina@math.vsu.ru