Asymptotic Solution of Linear-Quadratic Problems with Discontinuous Coefficients and Cheap Control

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Abstract: We consider a quadratic functional with a cheap control

$$J_\varepsilon(u) = \frac{1}{2}\sum_{j=1}^{2}\int_{t_{j-1}}^{t_j}\left(\left\langle \overset{(j)}{x}, \overset{(j)}{W}(t,\varepsilon)\overset{(j)}{x}\right\rangle + \varepsilon^2\left\langle \overset{(j)}{u}, \overset{(j)}{R}(t,\varepsilon)\overset{(j)}{u}\right\rangle\right)dt$$

to be minimized with respect to trajectories of a linear system

$$\frac{d\overset{(j)}{x}}{dt} = \overset{(j)}{A}(t,\varepsilon)\overset{(j)}{x} + \overset{(j)}{B}(t,\varepsilon)\overset{(j)}{u},\ t \in [t_{j-1}, t_j],\ j = 1, 2,$$

$$\overset{(1)}{x}(0,\varepsilon) = x^0, \quad \overset{(2)}{x}(t_1,\varepsilon) = \overset{(1)}{x}(t_1,\varepsilon).$$

Here $t \in [0, T]$, $0 = t_0 < t_1 < t_2 = T$, $t_j (j = 0, 1, 2)$ are fixed; $\varepsilon > 0$ is a small parameter; $\overset{(j)}{x} = \overset{(j)}{x}(t,\varepsilon) \in \mathbf{R}^m$, $\overset{(j)}{u} = \overset{(j)}{u}(t,\varepsilon) \in \mathbf{R}^r$; the matrices $\overset{(j)}{W}(t,\varepsilon)$, $\overset{(j)}{R}(t,\varepsilon)$ are symmetric and $\overset{(j)}{W}(t,0)$, $\overset{(j)}{R}(t,0)$ are positive definite; the matrices $\overset{(j)}{B}(t,0)$ are invertible, all matrix-functions are sufficiently smooth for all $t \in [t_{j-1}, t_j]$, $j = 1, 2$; $< \cdot, \cdot >$ denotes an inner product in an appropriate space.

Using a change of variables, we reduce the considered problem to a linear-quadratic problem with a singularly perturbed state equation in the critical case. Note that trajectories of the obtained state equation are discontinuous functions in the general case.

For constructing the asymptotic solution to the obtained problem, we substitute a postulated asymptotic expansion with boundary functions for the solution into the transformed problem condition and then define four types of optimal control problems to get the asymptotic terms. We show that a minimized functional is non-increasing if higher-order approximations to the optimal control are used. The estimates of the proximity of an approximate asymptotic solutions to the exact one are given.

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