Nitsche's Method for a Mass Transport Problem in Two-phase Incompressible Flows

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Abstract: Let $\Omega \subset \mathbb{R}^d$, $d = 2, 3$, be a convex polygonal domain that contains two different immiscible incompressible phases. The (in general time dependent) subdomains containing the two phases are denoted by Ω_1, Ω_2, with $\bar{\Omega} = \bar{\Omega}_1 \cup \bar{\Omega}_2$. A typical example is a droplet surrounded by another fluid. In this paper we only consider the *stationary* case in which the interface $\Gamma := \bar{\Omega}_1 \cap \bar{\Omega}_2$ does not depend on time. The fluid dynamics in such a flow problem is usually modeled by the incompressible Navier-Stokes equations combined with suitable conditions at the interface which describe the effect of surface tension. By $\mathbf{w}$ we denote the velocity field resulting from these Navier-Stokes equations. We consider a model which describes the transport of a dissolved species in such a two-phase flow problem. In strong formulation this model is as follows:

$$\frac{\partial u}{\partial t} + \mathbf{w} \cdot \nabla u - \operatorname{div}(\alpha \nabla u) = f \quad \text{in } \Omega_i,\ i = 1, 2,\ t \in [0, T], \tag{1}$$

$$[\alpha \nabla u \cdot \mathbf{n}]_\Gamma = 0, \tag{2}$$

$$[\beta u]_\Gamma = 0. \tag{3}$$

Here $\mathbf{n}$ denotes the unit normal at Γ pointing from Ω_1 into Ω_2. For a sufficiently smooth function v, $[v] = [v]_\Gamma$ denotes the jump of v across Γ. The interface condition in (2) results from the conservation of mass principle. The condition in (3) is the so-called *Henry condition*. The coefficients α and β are assumed to be positive and piecewise constant. In general, the solution u is *discontinuous across the interface*. For the spatial discretization of this problem we use an XFEM approach (with linear finite elements) and combine this with the Nitsche technique in order to satisfy the Henry condition (3) in a weak sense. For stationary elliptic interface problems this method is investigated in the literature by Hansbo. We present an error analysis for the method applied to the transport problem (1)-(3) and show results of numerical experiments.

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