On Worst-Case Robust Optimal Control of PDEs

R. Herzog[1] **and F. Schmidt**[1]

Abstract: Nearly every real world optimization problem depends on parameters p, which leads to a family of problems like

$$\min_u \ f(u,p).$$

Often, if the parameter p is not known exactly (i.e. it is uncertain), it is replaced by an estimate of its nominal value. In contrast to this we consider the worst-case problem

$$\min_u \quad \max_{p\in P_{ad}(u)} \ f(u,p)$$

where the parameter p can vary in a set of possible parameters $P_{ad}(u)$, which may depend on the upper-level variable u. An optimizer u of this problem is called a worst-case robust minimizer.

Sufficient conditions ensuring weak lower semi-continuity of the optimal value function

$$\varphi(u) = \max_{p\in P_{ad}(u)} \ f(u,p)$$

are presented. To this end, refined inner semi-continuity properties of set-valued maps are introduced which meet the needs of the weak topology in Banach spaces. The results are applied to prove the existence of solutions to various worst-case robust optimal control problems governed by semilinear elliptic partial differential equations.

As an outlook, we point out some of the difficulties associated with the derivation of first-order necessary optimality conditions. Moreover, we review the αBB-method for the solution of the lower-level problem.

[1] Department of Mathematics
Chemnitz University of Technology
09107 Chemnitz, Germany
{*roland.herzog, frank.schmidt*}*@mathematik.tu-chemnitz.de*