

# Network Pricing and Bilevel Optimization

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**Abstract:** The paradigm of pricing, either for improving the performance of infrastructures, or for maximizing the revenue of a private firm, pervades the economics literature. In the present talk, we consider the problem faced by a highway manager that seeks to maximize the revenue raised from tolls set on a network, while anticipating that users will travel on paths that maximize their individual utilities. This situation is closely related to the problem known as product line pricing, which is challenging from both the theoretical and computational points of view.

Some years ago, it was recognized that the network pricing problem fits the framework of bilevel programming, a branch of optimization concerned with the solution of nonconvex programs involving two noncooperative agents, and that is akin to a leader - follower, or Stackelberg, game. This approach led to studies that focused on the combinatorial nature of network pricing, either in its original formulation or variants thereof.

In the present talk, we consider a variant of the problem where all roads controlled by an authority are connected and form a path, as occurs in a toll highway. Assuming that tolls are levied with respect to all possible combinations of entry and exit points on the highway, one may focus on networks where a virtual arc is created for each entry - exit combination, and thus form an inner clique. Shortest paths that do not go through the highway are represented by arcs linking the various origins and destinations, and form an outer clique. Our aim is to provide a better understanding of the Clique Pricing Problem and to present algorithmic tools that can be transposed to situations arising in the field of revenue management. More precisely, we are interested in the complexity and the polyhedral structure of this specific Network Pricing Problem.

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