Convergence Rates for Tikhonov Regularization of Coefficient Identification Problems in Elliptic Equations

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Abstract: We investigate the convergence rates for Tikhonov regularization of the problem of identifying the coefficient q in the Neumann problem for the elliptic equation

$$-\text{div }(q\nabla u) = f \text{ in } \Omega, \quad q\frac{\partial u}{\partial n} = g \text{ on } \partial\Omega$$

when u is imprecisely given by z^δ in $\Omega \in I\!R^d, d \geq 1$. We regularize the problem by minimizing one of the following convex functionals

1) $\displaystyle \frac{1}{2}\int_\Omega q|\nabla(u(q) - z^\delta)|^2 dx + \rho\|q - q^*\|^2_{L^2(\Omega)}$,

2) $\displaystyle \frac{1}{2}\int_\Omega q|\nabla(u(q) - z^\delta)|^2 dx + \rho\int_\Omega |\nabla q|$,

3) $\displaystyle \frac{1}{2}\int_\Omega q|\nabla(u(q) - z^\delta)|^2 dx + \rho\Big(\int_\Omega |\nabla q| + \frac{1}{2}\|q\|^2_{L^2(\Omega)}\Big)$,

over the admissible set, where $u(q)$ is the solution of the Neumann boundary value problem, $\rho > 0$ is the regularization parameter, q^* is a guess for the sought coefficient. Taking the solutions of these optimization problems as the regularized solutions to the identification problem, we obtain the convergence rates of them correspondingly to 1) q^*-minimum norm solution, 2) a total variation-minimizing solution in the sense of the Bregman distance, or 3) a total variation-minimizing solution both in the sense of Bregman distance and L^2-norm under relatively simple source conditions without the smallness requirement on the source functions.

Similar results are obtained for the problem of identifying the coefficient a in the Neumann problem for the elliptic equation $-\Delta u + au = f$ in Ω, $\partial u/\partial n = g$ on $\partial\Omega$. Some numerical results are presented.

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