On Mixed-Integer Optimal Control of Partial Differential Equations

F. M. Hante[1] and S. Sager[1]

Abstract: We consider integer-restricted control of systems governed by abstract linear evolution equations. In particular, we address the problem of optimal control design for certain distributed parameter systems endowed with multiple actuators, where the task is to minimize costs associated with the dynamics of the system by choosing one of the actuators together with an ordinary control for each instant in time.

The mixed-integer program resulting from full discretization of such dynamical optimization problems is by its nature large-scale and computationally very expensive. As an alternative approach that is used successfully in context of mixed-integer optimal control problems for ordinary differential equations we consider relaxation methods.

Our analysis of the problem on the abstract level yields sufficient conditions such that the solution of the relaxed problem can be approximated with arbitrary precision by a solution satisfying the integer restrictions. The results are obtained by semigroup theory methods. As a direct consequence of a weak approximation principle for bounded functions, the linear dependency of the approximation error on the maximal step-size of the chosen control discretization in time known for ordinary differential equations generalizes to such infinite-dimensional settings. Moreover, the approximation procedure is constructive and gives rise to a convergent numerical method.

The analysis is supplemented with numerical experiments. As an example motivated by thermal manufacturing, we consider the optimal control of a two-dimensional heat equation with spatial scheduling of different actuators.

[1] Interdisciplinary Center for Scientific Computing
University of Heidelberg
Im Neuenheimer Feld 368, 69120 Heidelberg, Germany
{*falk.hante, sebastian.sager*}*@iwr.uni-heidelberg.de*