

# Splitting Methods for Semilinear Reaction-diffusion Equations

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**Abstract:** Splitting methods form an important class of competitive time integration schemes for evolution equations. Despite of their extensive use in real-life applications, the convergence properties of splitting methods for partial differential equations are still far from being fully understood. In this talk, we will concentrate on semilinear reaction-diffusion equations. Splitting the nonlinear reaction terms from the linear diffusion operator gives rise to attractive numerical schemes: the semiflow generated by the diffusion operator can be obtained by standard fast solvers (relying on fast Fourier transform techniques or fast Poisson solvers); the nonlinear part being local only requires the solution of an ordinary differential equation.

Our analysis is performed in an abstract Banach space setting of analytic semigroups and their related  $\varphi$ -functions. This framework allows us to analyze the stability and convergence properties of operator splitting methods and their spatial discretizations. In particular, the exponential Marchuk–Strang splitting turns out to be stable and second-order convergent. Moreover, it is shown that exponential operator splitting methods will preserve positivity of the solution in certain situations.

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