The Certified Reduced Basis Method for Saddle Point Problems

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Abstract: We present a framework for certified reduced basis approximation for the solution of parametrized partial differential equations, focusing on saddle point problems. The reduced basis method has two esssential components: (i) rapidly, uniformly convergent reduced-basis approximations, and (ii) associated rigorous *a posteriori* error bounds. In both components we exploit the affine parametric structure of the PDE to develop offline-online computational decompositions which in turn provide real-time response.

Although RB methods are well-developed for several classes of partial differential equations, parametrized saddle point problems pose additional difficulties. In particular, parameter-dependent constraints cause complications not only in the choice of stable reduced basis approximation spaces but also in the construction of rigorous and computationally efficient *a posteriori* error bounds.

First, we discuss the issue of approximation stability, exploring ways to efficiently enrich the velocity approximation space to ensure stability as well as rapid convergence. Second, we present and compare several approaches to *a posteriori* error estimation. The first approach, based on a penalty approximation, allows the computation of *a posteriori* error estimates which circumvent the computation of expensive inf-sup constants. However, this is achieved at the expense of an additional error in the truth finite element approxmation. The second approach is a direct application of Brezzis theory for saddle point problems, and enables computation of separate error bounds for the velocity and the pressure. We summarize the advantages and disadvantages of these two approaches, particularly when compared with already existing approaches using Babuska's theory for noncoercive problems. We present numerical results for a problem of Stokes flow through a two-dimensional channel with a parametrized rectangular obstacle. The results demonstrate the significant effects of the enrichment of the velocity space on approximation stability and convergence, and the performance of the proposed error bounds with respect to sharpness and computational cost.

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