An ADI Orthogonal Spline Collocation Method for Nonlinear Reaction-Diffusion Systems

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Abstract: A new numerical method is described for the approximate solution of a class of two-component nonlinear reaction-diffusion systems in a rectangle. It combines a spatial discretization based on orthogonal spline collocation (OSC) with C^1 piecewise polynomials of degree ≥ 3 with an alternating direction implicit (ADI) method to advance in time. Based on an extrapolated Crank-Nicolson OSC method, the method is algebraically linear. Like all ADI OSC methods, it reduces the two-dimensional problem to sets of independent OSC two-point boundary value problems in each coordinate direction at each time step. With standard choices of bases for the spline space employed in the OSC discretization, these problems give rise to almost block diagonal linear systems, which can be solved efficiently using existing software at a total cost of $O(\mathcal{N})$ operations, where $\mathcal{N}$ is the number of unknowns. The efficacy of the method is demonstrated on the solution of well-known examples of reaction-diffusion models arising in chemistry and biology, specifically the Brusselator, Gray-Scott, Gierer-Meinhardt and Schnakenberg models, and comparisons are made with other numerical techniques considered in the literature. Moreover, the method is shown to produce approximations which are of optimal global accuracy in various norms, and to possess superconvergence properties at the nodes of the partition of the spatial domain on which the spline space is defined.

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