On Penalty and Gap Function Methods for Bilevel Equilibrium Problems

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Abstract: Let C be a nonempty closed convex subset in $\mathbb{R}^n$ and $f, g : C \times C \to \mathbb{R}$ be two bifunctions satisfying $f(x,x) = g(x,x) = 0$ for every $x \in C$. Such a bifunction is called an equilibrium bifunction. We consider the following bilevel equilibrium problem:

$$\text{Find } \bar{x} \in S_g \text{ such that } f(\bar{x}, y) \geq 0, \forall y \in S_g, \tag{1}$$

where $S_g = \{u \in C : g(u,y) \geq 0, \forall y \in C\}$, i.e., S_g is the solution set of the equilibrium problem

$$\text{Find } u \in C \text{ such that } g(u,y) \geq 0, \forall y \in C. \tag{2}$$

In this talk, we consider bilevel pseudomonotone equilibrium problems of the form (1). First, we propose a penalty function method to convert the problem (1) into one-level ones. Next, we use a regularized gap function for solving the penalized problems. We generalize a pseudo ∇-monotonicity concept from ∇-monotonicity and prove that under certain pseudo ∇-monotonicity properties, any stationary point of a regularized gap function on the convex set C is a solution to the penalized subproblem. Finally, as an application, we discuss a special case that arises from the Tikhonov regularization method for pseudo monotone equilibrium problems.

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