Stabilization of Unstable Equilibrium States in Magnetohydrodynamics

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Abstract: The motion of incompressible viscous and conducting fluid in a bounded domain $\Omega \subset \mathbb{R}^3$ with boundary Γ is modeled in terms of dimensionless variables by MHD equations:

$$\partial u/\partial t - \nu \Delta u + (u\nabla)u = -\nabla p + a \cdot \operatorname{rot} B \times B,\ x \in \Omega,\ t \in (0,T), \tag{1}$$

$$\partial B/\partial t + \operatorname{rot} E = 0,\ j = \operatorname{rot} B = 1/\nu_\mu (E + u \times B) + J_c,\ \operatorname{div} u = 0,\ \ \operatorname{div} B = 0. \tag{2}$$

Here u, B, E and j are the vector fields of velocity, magnetic induction, electric intensity and current density respectively; p is the flow pressure, $\nu = 1/Re$, $\nu_\mu = 1/R_\mu$, $a = M^2/Re\,R_\mu$, where Re,Re_μ and M are the Reynolds number, Reynolds magnetic number and Hartmann number. The vector field of external currents J_c is a control. To the equations (1)–(2) we add the initial and the boundary value conditions

$$u|_{t=0} = u_0(x),\ B|_{t=0} = B_0(x),\ x \in \Omega,\ \ u|_\Gamma = 0,\ B \cdot n|_\Gamma = 0,\ n \times E|_\Gamma = 0, \tag{3}$$

where n is the unit outward normal to the boundary. The equilibrium configuration of a conducting liquid is determined by equations of magnetic hydrostatics,

$$\nabla p_s = a \cdot \operatorname{rot} B_s \times B_s,\ \operatorname{rot} E_s = 0,\ \ \operatorname{rot} B_s = 1/\nu_\mu E_s + J_s,\ \ \operatorname{div} B_s = 0,\ \ x \in \Omega.$$

Stabilization problem is the following: *Find a feedback control operator $J_c = \Lambda(B)$ such that the solution $\{u, B\}$ of a closed system (1)–(3) with control J_c exponentially converges to the equilibrium configuration $\{u_s = 0, B_s\}$,*

$$\int_\Omega u^2(x,t)\,dx + \int_\Omega (B(x,t) - B_s(x))^2\,dx \le C\mathrm{e}^{-\sigma t}, \quad \textit{where } C, \sigma > 0,\ t \to +\infty.$$

The construction of stabilizing control is based on spectral properties of operators simulating dissipative and viscous terms in the model of MHD. The main result is to construct a stabilizing operator Λ, such that the equilibrium configuration is stable singular point of the dynamical system generated by the equations (1)–(2) in the corresponding phase space and the operator Λ has finite-dimensional image, lying in the ball of given radius.

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