New Cutting Planes for the Shortest Path Routing Polytope

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Abstract: Routing in current communication networks is often based on shortest path routing protocols, such as OSPF or IS-IS. Shortest path routing problems arise as subproblems of many network optimization problems, and a frequent question is to decide if for a given set of end-to-end paths in a directed graph there exist arc weights, such that they form unique shortest paths to their respective destinations. In this paper, we study the Unique Shortest Path Systems Problem with two root nodes (2-USPS): Given a complete directed graph $G = (N, A)$ and two spanning arborescenses, A^{l_0} and A^{l_1}, with roots $l_0, l_1 \in N$, is it possible to find arc weights $w \in \mathbf{Z}_+^A$, such that A^{l_0} and A^{l_1} simultaneously are the shortest path arborescenses to l_0 and l_1, respectively? Specifically, we consider the polytope that is the convex hull of all yes-instances to this question, i.e. the polytope whose extreme points are the incidence vectors of pairs (A^{l_0}, A^{l_1}) of arborescences that can be realized simultaneously as shortest path arborescences. We derive facet-defining inequalities for this polytope that stem from a structure called a $valid\ cycle$, whose presence is known to be a sufficient condition for the infeasibility of a system of end-to-end paths as shortest paths.

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