# Feedback Control of Hybrid PDE Systems with Applications to the Operation of Energy Efficient Buildings

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**Abstract:** The problems of designing and controlling energy efficient building systems are naturally described as distributed parameter optimization and control problems with boundary inputs. Moreover, new low energy concepts such as chilled beams and radiant heating leads to problems with Dirichlet, Neumann and Robin type boundary conditions. The physics of displacement ventilation and buoyancy-driven flows are modeled by the Boussinesq approximations. In particular, let $\Omega \subset \mathbf{R^d}$, $d = 2, 3$, be an open smooth bounded and connected domain with a regular boundary $\Gamma$ of class $C^2$. We consider a control problem for the Boussinesq equations

$$\begin{aligned}
\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}, &= \frac{1}{Re}\Delta\mathbf{v} - \nabla p + \bar{\mathbf{e}}\frac{G_r}{Re^2}\theta + f_{\mathbf{v}} \\
\text{div } \mathbf{v} &= 0, \\
\frac{\partial \theta}{\partial t} + \mathbf{v}\cdot\nabla\theta &= \frac{1}{RePr}\Delta\theta + f_\theta, \\
B_{\mathbf{v}}\mathbf{v}|_\Gamma = c_{\mathbf{v}}(x)u_{\mathbf{v}}, &\qquad B_\theta\theta|_\Gamma = c_\theta(x)u_\theta,
\end{aligned}$$

where $\mathbf{v}(t,x)$ is the velocity, $p(t,x)$ is the pressure and $\theta(t,x)$ is the fluid temperature. Here, $\bar{\mathbf{e}}$ is the gravitational force direction, $f_{\mathbf{v}}$ is an external body force and $f_\theta$ is a heat source density. The functions $c_{\mathbf{v}}(x)$ and $c_\theta(x)$ are given functions defined on $\Gamma$, while $B_{\mathbf{v}}$ and $B_\theta$ are boundary operators through which the controls are introduced on the boundary. Because of the application, we consider the case where there are only a finite number of control inputs so that the control space is $\mathcal{U} = \mathbf{R}^m$. In addition, these control inputs are usually the output of a HVAC (Heating, Ventilation and Air Conditioning) system which is modeled as an "actuator". We discuss a LQR type control problem for this system with Robin/Neumann boundary conditions and apply the results to a 2D problem to illustrate the ideas and demonstrate the computational algorithms.

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