Numerical Computation of Robust Controllers for Parabolic Systems

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Abstract: We will consider the computation of a robust feedback controller for systems described by parabolic PDEs. Robustness is to be understood in the sense that the controller will stabilize not only the nominal system, but achieves this also under perturbations caused by external disturbances or unmodeled dynamics. One way to achieve this goal is H_∞ control. In this approach, a controller is computed which minimizes the worst-case error transfer from exogenous inputs to the measurement error. The usual approach to compute an H_∞-controller is based on the so-called γ-iteration, which determines the infimum of the attainable H_∞-norm error for the associated transfer function. This iteration requires in each step the solution of two dual operator Riccati equations and to determine the spectral radius of the product of their solutions.

If this concept is applied to a system described by a parabolic partial differential equation, one needs to discretize the PDE in space and to run the γ-iteration using the resulting finite-dimensional algebraic Riccati equations (AREs) and the associated product spectral radius as approximations. It is known that for a fine-enough discretization level, the obtained finite-dimensional controller robustly stabilizes the infinite-dimensional system [K. Ito, *SIAM J. Cont. Optim.* (1990), 28:1251–1269; K. Morris, *J. Math. Syst., Estim., and Control* (1994), 4:1–30]. The resulting AREs are usually large-scale due to the fine discretization required, and are defined by sparse and low-rank coefficient matrices. The efficient numerical solution of large-scale AREs arising in LQR or LQG design has made significant progress in recent years. Common to the successful methods is the use of Newton's method for AREs and the re-writing of the method in terms of (approximate) low-rank factors of the Newton iterates. These approaches fail to be applicable to the AREs resulting from H_∞-control: there, the quadratic terms of the AREs are often indefinite, in other words, the Hessian of the Riccati function is indefinite so that in general, Newton's method will not converge. In order to overcome this difficulty, several authors have suggested iterative methods which are intended for small-scale systems and which are based on sequences of Lyapunov equations similarly to Newton's method. Here, we will discuss how these methods can be used to solve large-scale problems. For this purpose, we show how the methods can be re-formulated in terms of low-rank factors of the iterates and how these can be computed using recent ideas from numerical linear algebra.

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