# Bridges from ODE Optimal Control to PDE Optimal Control

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**Abstract:** The well-known Bryson-Denham-Dreyfus indirect adjoining method [3] has turned out to be the most efficient way for solving state-constrained optimal control problems with ordinary differential equations in practice. In the talk it will be shown how this approach can be adopted to elliptic optimal control problems. The benefits are twofold: We obtain new necessary conditions with Lagrange multiplier having higher regularity [1] compared to the direct adjoining approach [4], [5] as in ODE optimal control and the resulting numerical method for solving a new class of set optimal control problems by shape linearization [2] does not need any regularization techniques and can be described in function spaces. Numerical results — so far for academic problems only — show the efficiency of the new numerical method.

## References

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