A Theoretical Analysis of the Condition of Mathematical Programs Resulting from Direct Shooting Approaches

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Abstract: Methods for the solution of dynamic optimization problems by numerical approximation can be divided into two classes, i.e. simultaneous methods, and methods based on the solution of an initial value problem (IVP), as for instance single shooting or multiple shooting. Single shooting relies on the solution of a single IVP and the only optimization variables are stemming from control vector parameterization and time-invariant parameters. Multiple shooting divides the time horizon into several shorter stages, where the dynamic model is solved by a number of independent IVPs on each stage. The drawback for both shooting approaches is that the local IVPs used on each subinterval may be ill-conditioned, e.g. if the underlying ordinary differential equation (ODE) system admits unstable modes. An ill-conditioned ODE system may cause severe problems for the numerical integrator when solving the IVP or for the optimizer when solving the mathematical program. The latter problem results from the embedded IVP in the shooting algorithms, which influences the condition of the mathematical program itself.

A problem is well-conditioned if a small change in the *data* leads to a small change in the *results*. Hence, the gradients of the state variables w.r.t the decision variables influence the condition of the mathematical program.

In this contribution, we study the influence of the condition of the IVP on the condition of the mathematical program resulting from the shooting approaches. The basic ideas and theoretical concepts are derived for unconstrained dynamic optimization problems with an underlying linear system. An illustrative simple example is chosen to give insight on the condition of the mathematical program and its effect on the solution of the dynamic optimization problem.

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