Necessary Optimality Conditions and Approximations for Infinite-Horizon Optimal Control

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Abstract: The talk revisits the issue of necessary optimality conditions for infinite-horizon optimal control problems. It is proved that the normal form maximum principle holds with an explicitly specified adjoint variable if an appropriate relation (called $dominating\ discount\ condition$) between the discount rate, the growth rate of the solution and the growth rate of the objective function is satisfied. The main novelty is that the result applies to general non-stationary systems and the optimal objective value needs not be finite (in which case the concept of overtaking optimality is employed).

If the state-derivative of the objective integrand is bounded and the system has globally the so-called one-sided Lipschitz property, then one can reformulate the dominating discount condition as $\rho > \lambda$, where ρ is the discount rate and λ is the one-sided Lipschitz constant (which may be negative). In this case it is shown that the current-value adjoint variable for which the normal form maximum principle holds is characterized as the unique bounded solution of the adjoint equation.

The above results are applicable to several economic models for which the known optimality conditions fail. An example will be given.

The approach utilized in the proof involves approximations of the infinite-horizon problem by problems on a finite horizon. The dominating discount condition allows to address the issue of numerical approximations of infinite-horizon problems by approximate solutions (involving also discretization) of finite-horizon problems. Results of this type will be presented and will relate our talk to the mainstream of the conference.

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