Chaotic Regularization

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Abstract: The integration to steady state of many initial value ODEs and PDEs using the forward Euler method can alternatively be considered as gradient descent for an associated minimization problem. Greedy algorithms such as steepest descent for determining the step size are as slow to reach steady state as is forward Euler integration with the best uniform step size. Yet other, much faster gradient descent methods using bolder step size selection exist. They can be preferable to conjugate gradients when matrix-vector multiplications are performed inaccurately. These faster gradient descent methods, however, yield chaotic dynamical systems for the iteration residuals.

The steepest descent method is also known for the regularizing or smoothing effect that the first few steps have for certain inverse problems, amounting to a finite time regularization. We further investigate using the faster gradient descent variants for this purpose in the context of denoising and deblurring of images and also of problems involving data inversion for elliptic PDEs. When the combination of regularization and accuracy requirements demands more than about a dozen steepest descent steps, the alternatives offer an advantage, even though (indeed because) the absolute stability limit of forward Euler is carefully yet severely violated.

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