Parallel $CQ-$ Methods for a Finite Family of Relatively Nonexpansive Mappings

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Abstract: Various problems of science and engineering, such as the convex feasibility problems with applications in optimization theory, image processing, radiation therapy treatment planning, etc..., can be reduced to a problem of finding a common fixed point of a family of nonexpansive mappings.

In 2005, Matsushita and Takahashi proposed the so-called $CQ-$ method for finding a fixed point of a relatively nonexpansive mapping. Several attempts to generalize the $CQ-$ method for finding a common fixed point of a finite or infinite family of (relatively) nonexpansive mappings have recently made by Takahashi and Zembayashi, Plubtieng and Ungchittrakool, Su, Wang and Xu, Cholamjiak, Suantai, etc... Very recently, X.F. Liu (*Viet. J. Math.* 39(1) 2011, 63-69) proposed a cyclic $CQ-$ method for a finite family of N relatively nonexpansive mappings.

In this report we propose a parallel $CQ-$ method, which may be considered as a counterpart of a cyclic (sequential) $CQ-$ method for finding a common fixed point of a finite family of relatively nonexpansive mappings. Based on the parallel computation we can reduce the overall computational effort under commonly used conditions on the spaces and mappings. Moreover, the additional computation cost of our method is essentially negligible. Further, for mappings acting in a real Hilbert space, we modify the proposed algorithm such that all the iterates can be computed effectively. A similar idea was employed in our recent work on a parallel version of Solodov-Svaiter's method for solving a system of monotone equations. Finally we perform both parallel and sequential $CQ-$ methods for finding a common fixed point of two nonlinear nonexpansive integral operators and show the effectiveness of the parallel $CQ-$ method compared to the cyclic one.

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