Discontinuous Galerkin Methods for the Helmholtz Equation with Large Wave Number

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Abstract: This paper develops and analyzes some interior penalty discontinuous Galerkin methods using piecewise linear polynomials for the Helmholtz equation with the first order absorbing boundary condition in the two and three dimensions. It is proved that the proposed discontinuous Galerkin methods are stable (hence well-posed) without any mesh constraint. For each fixed wave number k, optimal order (with respect to h) error estimate in the broken H^1-norm and sub-optimal order estimate in the L^2-norm are derived without any mesh constraint. The latter estimate improves to optimal order when the mesh size h is restricted to the preasymptotic regime (i.e., k^2h is not small). Numerical experiments are also presented to gauge the theoretical result and to numerically examine the pollution effect (with respect to k) in the error bounds. The novelties of the proposed interior penalty discontinuous Galerkin methods include: first, the methods penalize not only the jumps of the function values across the element edges but also the jumps of the normal and tangential derivatives; second, the penalty parameters are taken as complex numbers of positive imaginary parts so essentially and practically no constraint is imposed on the penalty parameters.

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