

# Solving Polynomial Optimization Problems via the Truncated Tangency Variety and Sums of Squares

H. H. Vui<sup>1</sup> and P. T. Son<sup>2</sup>

**Abstract:** This paper proposes a method for finding the global infimum of a polynomial  $f$  on a semialgebraic set  $S$  via sum of squares relaxation over its truncated tangency variety, even in the case where the polynomial  $f$  does not attain its infimum on  $S$ : Under a constraint qualification condition, it is demonstrated that: (i) The infimum of  $f$  on  $S$  and on its truncated tangency variety coincide; and (ii) A sums of squares certificate for nonnegativity of  $f$  on its truncated tangency variety. These facts imply that we can find a natural sequence of semidefinite programs whose optimal values converges monotonically increasing to the infimum of  $f$  on  $S$ .

---

<sup>1</sup> Institute of Mathematics  
Vietnamese Academy of Science and Technology  
18 Hoang Quoc Viet Road, Cau Giay District  
10307 Hanoi, Vietnam  
[hhvui@math.ac.vn](mailto:hhvui@math.ac.vn)

<sup>2</sup> Department of Mathematics, University of Da Lat  
Da Lat, Vietnam  
[pham\\_ts@yahoo.co.uk](mailto:pham_ts@yahoo.co.uk)