

Non-Smooth Dynamical Systems and Variational Inequalities: An Overview

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Abstract: It is well known that many processes in automatic regulation, physics, mechanics, biology, economy, ecology, etc., can be modelled by dynamical systems. Many stability results in the theory of dynamical systems and their applications were obtained by construction of appropriate Lyapunov functionals.

There are many systems which have non-smooth dynamics. Examples include systems with Coulomb friction, contact interactions and variable structure systems where control inputs are allowed to be discontinuous. It is essential to rigorously analyze these systems and address such issues as the existence of equilibria, their stability, and qualitative dynamics.

In extending the technique of Lyapunov functions to control systems, a number of new issues arise. To begin with, we can distinguish two cases: we may require that all trajectories go to the origin (strong stability) or that (for a suitable choice of the control function) some trajectory goes to zero (weak stability, or controllability). In the latter case, unlike the former, it turns out that characterizing stability in terms of smooth Lyapunov functions is not possible; thus elements of nonsmooth analysis become essential.

The aim of this talk is to extend some known stability results for differential inclusions to the case of non-smooth Lyapunov function.

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