Optimal Control of a Hybrid Investment Model

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Abstract: In this talk we present a proof of Pontryagin's Maximum Principle for a simple Hybrid Optimal Control Problem and investigate a Hybrid Investment Model.

The hybridity in our problem just means the presence of a finite number k of different dynamics and furthermore the restriction to choose only one of these dynamics in every time point. To describe this integer character we define the set $Z_0^k([t_0,t_1])$ of all piecewise partitions of order k of the time interval $[t_0,t_1]$ and identify the elements $\mathcal{A}=(\mathcal{A}_1,...,\mathcal{A}_k)\in Z_0^k([t_0,t_1])$ by the characteristic vectorfunctions

$$\chi_{\mathcal{A}}(t)=\big(\chi_{\mathcal{A}_1}(t),...,\chi_{\mathcal{A}_k}(t)\big).$$

Hence we discuss the following problem

$$\begin{aligned}
&J\big(x(\cdot),u(\cdot),\mathcal{A}\big)=\int_{t_0}^{t_1} f\big(t,x(t),u(t),\chi_{\mathcal{A}}(t)\big)dt\to\inf!\\
&\dot{x}(t)=\varphi\big(t,x(t),u(t),\chi_{\mathcal{A}}(t)\big),\quad x(t_0)=x_0,\\
&u(\cdot)\in PC\big([t_0,t_1],U\big),\quad \mathcal{A}\in Z_0^k([t_0,t_1]),
\end{aligned}$$

with the state $x(\cdot)$, the control $u(\cdot)$ and the switching variable $\mathcal{A}$.

After the theoretical preparations we consider the investment model

$$\begin{aligned}
&J\big(x(\cdot),u(\cdot),\mathcal{A}\big)=\int_0^T f\big(t,x(t),u(t),\chi_{\mathcal{A}}(t)\big)dt=\int_0^T f_{1/2}\big(t,x(t),u(t)\big)dt\to\sup!\\
&\dot{x}(t)=\varphi\big(t,x(t),u(t),\chi_{\mathcal{A}}(t)\big)=\varphi_{1/2}\big(t,x(t),u(t)\big),\quad x(0)=x_0>0,\\
&u(\cdot)\in PC\big([0,T],[0,1]\big),\quad \mathcal{A}=(\mathcal{A}_1,\mathcal{A}_2)\in Z_0^2([0,T]),
\end{aligned}$$

with

$$\begin{aligned}
&f_1\big(t,x(t),u(t)\big)=\big(1-u(t)\big)\cdot x(t), &&\varphi_1\big(t,x(t),u(t)\big)=u(t)\cdot x(t), && t\in\mathcal{A}_1,\\
&f_2\big(t,x(t),u(t)\big)=\big(1-u(t)\big)\cdot x^\alpha(t), &&\varphi_2\big(t,x(t),u(t)\big)=u(t)\cdot x^\alpha(t), && t\in\mathcal{A}_2,
\end{aligned}$$

and

$$\alpha\in(0,1)\text{ constant},\quad T>\max\left\{1;\frac{x_0^{1-\alpha}}{\alpha}\right\}\text{ fixed.}$$

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