Semidefinite Programming Relaxation for Constrained 0-1 Quadratic Programs

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Abstract: We investigate semidefinite programming (SDP) relaxations for linearly constrained 0-1 quadratic programs. Two classes of problems are considered: linear equality constrained 0-1 quadratic program and quadratic knapsack program.

We discuss the Lagrangian duality properties of linear equality constrained quadratic binary programming. An underestimation of the duality gap between the primal problem and its Lagrangian dual or SDP relaxation is derived using the distance from $\{-1, 1\}^n$ to certain affine subspace, while the computation of this distance can be achieved by the cell enumeration of hyperplane arrangement. Alternative Lagrangian dual schemes via the exact penalty and the squared norm constraint reformulations are also discussed.

For quadratic knapsack problem, we present optimality conditions for the primal and dual problem. It is shown that the SDP relaxation of quadratic knapsack problem does not always possess a unique optimal solution. This is in contrast with the unconstrained 0-1 quadratic problem where the SDP relaxation always has a unique optimal solution. We then derive a necessary and sufficient condition to ensure the uniqueness of the SDP relaxation solution for QKP. The duality gap between QKP and its SDP relaxation is analyzed using the distance from set $\{0, 1\}^n$ to certain polyhedral set. It is shown that the duality gap can be reduced by an amount proportional to the square of the distance.

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