Domain Decomposition Solution of Nonlinear Two-Dimensional Parabolic Problems by Random Trees

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Abstract: A domain decomposition method is developed for solving numerically nonlinear two-dimensional parabolic problems, based on the probabilistic representation of solutions as an average on suitable multiplicative functionals. Such a probabilistic representation requires generating suitable random trees, which play a similar role to that played by the stochastic processes in linear problems.

First, only few values of the sought solution inside the space-time domain are computed (by a Monte Carlo method on the trees), and a two-dimensional interpolation is then accomplished, in order to approximate interfacial values of the solution inside the domain. Finally, a fully decoupled set of sub-problems is obtained. This fact represents a definitely more advantageous circumstance, compared to what happens in any other existing deterministic domain decomposition methods.

The algorithm is suited to massively parallel implementation, enjoying arbitrary scalability and fault tolerance properties. Pruning the trees is shown to increase appreciably the efficiency. Numerical examples, including some for the KPP equation in 2D, are given. The performance of the PDD method is also assessed comparing our results with those obtained by competitive parallel numerical codes, broadly used in the high-performance scientific community.

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