# Higher Order Galerkin Discretizations for the Non-Stationary Navier Stokes Equations

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**Abstract:** In this work we will present efficient solution methods for the discretization of the non-stationary Navier Stokes equations applied to problems with moderate Reynolds number.

Both, for spatial and temporal discretization, higher order Galerkin methods are utilized. Space discretization is accomplished with standard elements of Taylor Hood type. Additional local projection terms are added to cope with dominant transport [1].

For time discretization the discontinuous Galerkin (dG) method is used. Super approximation results guarantee high approximation order in the time nodes. Due to good stability properties (strongly A-stable) dG-methods are well suited for flow problems.

Especially for the dG$(1)$-method a new and efficient iterative solution scheme is presented which allows to approximate the time steps without the need of solving coupled equations involving multiple states. Instead, simple one-step methods are employed.

For the solution of the arising systems of linear equations we carry on the works of Turek [2] to higher order finite elements. A fast iterative algorithm approximates the solution in the Schur complement form.

The use of Galerkin methods in space and time allows for error estimation with the dual weighted residual method by Becker & Rannacher [3] and for the easy embedding in optimization routines.

[1] M. Braack, E. Burman, *Local projection stabilization for the Oseen problem and its interpretation as a variational multiscale method*, SIAM J. Numer. Anal., 43 (6), pp. 2544-2566 (2006)

[2] S. Turek, *Efficient Solvers for Incompressible Flow Problems*, Lecture Notes in Computational Science and Engineering 6, Springer (1991)

[3] R. Becker, R. Rannacher, *An Optimal Control Approach to A Posteriori Error Estimation in Finite Element Methods*, Acta Numerica 2001, Cambridge University Press, pp. 1-225 (2001)

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