A Numerical Study for the Performance of DG methods Based on Different Numerical Fluxes

J. Qiu[1]

Abstract: Runge-Kutta discontinuous Galerkin (RKDG) method is a high order finite element method for solving hyperbolic conservation laws employing useful features from high resolution finite volume schemes, such as the exact or approximate Riemann solvers serving as numerical fluxes, TVD Runge-Kutta time discretizations, and limiters. In most of the RKDG papers in the literature, the Lax-Friedrichs numerical flux is used due to its simplicity, although there are many other numerical fluxes which could also be used. In this presentation we systematically investigate the performance of the RKDG method based on different numerical fluxes, including the first-order monotone fluxes such as the Godunov flux, the Engquist-Osher flux etc., and second-order TVD fluxes, with the objective of obtaining better performance by choosing suitable numerical fluxes.

Extensive one and two dimensional simulations on the hyperbolic systems of Euler equations indicate that RKDG methods with the LF flux cost the least CPU time among all, but the numerical errors and resolution of solutions on the discontinuities are also the worst among all. The RKDG methods with the Godunov or EO fluxes seem to cost significantly more CPU time than the RKDG-LF method. The HLL, HLLC and MUSTA fluxes might be good choices as fluxes for the RKDG method when all factors such as the cost of CPU time, numerical errors and resolution of discontinuities in the solution are considered.

We have also tested the RKDG methods based on a few other numerical fluxes, such as the second-order Lax-Wendroff (LW) flux, the Warming-Beam (WB) flux, and the WAF flux . Our numerical tests indicate that spurious oscillations appear for the Lax shock tube problem for the RKDG-LW and RKDG-WB schemes, and the codes are unstable (they blow up) for the blast wave test case. Because the WAF flux is based on the average of Godunov and Lax-Wendroff fluxes, it is more costly than the Godunov flux and hence is not comparable in CPU time cost with schemes such as RKDG-HLL and RKDG-MUSTA.

[1] Department of Mathematics, Nanjing University
22 Hankou Road, Nanjing, Jiangsu, China, 210093
jxqiu@nju.edu.cn