

# On Strong Uniform Optimal Solutions of Infinite Horizon Optimal Control Problems

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**Abstract:** Still at the beginning of the previous century the optimal control problems with infinite horizon became very important with regards to applications in economics, where an infinite horizon seems to be a very natural phenomenon. These problems were treated by many authors and various necessary, sufficient as well as transversality conditions were obtained. The problem we consider in this talk is formulated as follows. Minimize the functional

$$J(x, u) = \int_0^{\infty} r(t, x(t), u(t)) \tilde{\nu}(t) dt$$

subject to all pairs  $(x, u) \in W_p^{1,n}(\mathbb{R}^+, \nu) \times L_p^r(\mathbb{R}^+)$ , satisfying state equations, control restrictions and initial conditions. The integral in the functional  $J$  is understood in Lebesgue sense. The remarkable on this statement is the choice of the weighted Sobolev- and weighted Lebesgue spaces as state and control spaces respectively. We call functions  $\nu$  and  $\tilde{\nu}$  weights. These considerations give us the possibility to extend the admissible set and simultaneously to be sure that the adjoint variable belongs to a reflexive Banach space. For special economic models with infinite horizon we find the solution by proving sufficiency conditions. At the same time we construct the finite horizon model for which strong uniform optimality is preserved.

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