Parallel and Accurate Matrix Multiplication based on Optimized BLAS

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Abstract: Our talk is concerned with an accurate computation for dense-matrix multiplication by using floating-point arithmetic. Recently, we proposed a method which returns an accurate result for matrix multiplication. The input two matrices are divided into sum of matrices and the final result is computed by sum of the results of matrix multiplication. In this computations, rounding errors do not occur in many matrix multiplications which has relatively larger bits of the result. When we implement matrix computations, optimized BLAS (Basic Linear Algebra Subroutines, for example, Goto BLAS, Intel Math Kernel Library and ATLAS) is very convenient since the library is specialized for an architecture. Moreover, the routine for matrix multiplication automatically use multi-threads. Our method strongly depends on such a convenience BLAS so that high performance computing can be achieved easily.

In this talk, we show the efficiency of parallelization of the proposed method. Except for the cost of matrix multiplication, our method needs only $\mathrm{O}(n^2)$ computations (where n is dimension of matrix). The routine of matrix products in optimized BLAS achieves nearly peak performance and the efficiency of parallelization for our method is almost equal to that of optimized BLAS routine.

When we implement our algorithm, there is a drawback to the required amount of memory. We must prepare amount of memory for storing at least 5 matrices. We overcome this drawback by using block computations. When we divide the result into two-by-two blocks, the required amount of working memory becomes almost half. By applying this strategy, $\mathrm{O}(n^2)$ computations increase. However the performance of the proposed method still remains high.

Finally, we show numerical examples on shared memory systems to illustrate the effectiveness of the proposed method.

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