Fast Quasi-Quadruple Precision Computation of Sparse Matrix-Vector Product

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Abstract: Computing a sparse matrix-vector product is a basic task and plays an important role in scientific computing. On the other hand, the quality of the results obtained by floating-point arithmetic sometimes significantly suffers from the rounding errors. For example, most of iterative solutions methods for systems of linear equations such as CG (Conjugate Gradient), BICGSTAB and GMRES consist of sparse matrix-vector product(s), SAXPY (Scalar Alpha X Plus Y) operations and dot products. Due to the rounding errors occurring in these operations, such iterative solution methods sometimes become slow in converging or even do not converge well.

The purpose of this research is to develop a fast and high-precision algorithm of computing the matrix-vector product $y = Ax$ with A being a large sparse matrix and x a (dense) vector. In particular, we present some algorithms as if the results of sparse matrix-vector product $y \leftarrow Ax + \beta y$, SAXPY operation $y \leftarrow \alpha x + y$ and dot product $\gamma \leftarrow x^T y$ are computed in quasi-quadruple precision arithmetic, where $A \in \mathbb{F}^{n \times n}$, $x, y \in \mathbb{F}^n + \mathbb{F}^n$ and $\alpha, \beta, \gamma \in \mathbb{F} + \mathbb{F}$ for $\mathbb{F}$ being the set of IEEE 754 double precision floating-point numbers. The proposed algorithms are based on fast and efficient algorithm of calculating sum and dot product. Numerical results are also presented with applying the proposed algorithms to some iterative solution methods for systems of linear equations showing the performance of the proposed algorithms.

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