A Benders Decomposition Method for the Signal-Optimization Problem in Traffic Light

M. Minoux[1], **M. T. Ngo**[1,2], **and N. V. Hung**[1]

Abstract: Traffic congestion is one of the worst problems of public transport in many countries, it makes waste of fuel, travel time and also is one of the sources of environmental and socioeconomic problems. The coordination control of traffic in urban areas by signals is one of the most effective means of improving the fluidity of the traffic.

Given a traffic network with specified demands in the entries and directional distributions in each intersection, we consider the signal-optimization problem which tries to find the best offset values and sequences of "green/red" in each signal cycle minimizing the total traffic delay of all vehicles in network over a given time T.

In this paper, we introduce first a new mixed 0-1 formulation for this problem. The decision variables are traffic light and offset variables. We use a first order macroscopic traffic simulation model (CTM model) in our formulation to modelling the traffic. We introduce the concept of virtual cell in order to modelling all movements at intersections which is new in compare to existing approaches which often neglect the left-turning movement. Other basic phenomena of traffic such as shockwave and queue dynamic are also captured in the model.

We propose, in the second part, a Benders decomposition based method to solve the problem. In particular, we show that solving the subproblem in Benders decomposition, i.e. generating Benders cuts, can be done combinatorially using our simulation model. This result helps us avoid to use linear programming solver for subproblem solving and hence accelerate considerably the execution time of our code. Our method is very efficient as we can solve meshed instances containing upon 9 intersections within 2 heures which outperform (in size of network) all previous published works. The below table shows the advantage of our specialized method in compare to using simply the commercial solver CPLEX to solve the model.

Instance		Benders decomposition					CPLEX		
Network	Traffic	Sol init	Sol Best	*gap*	Ite	Time	*B&B*	gap	Time
3a	light	501.1	129.8	0%	15	3	26	0%	385
3a	saturated	1452.2	950.5	0%	22	4	58	0%	760
4a	light	1021.6	979.7	0%	130	38	331	0%	2871
4a	saturated	3375.2	2323.2	0%	88	22	162	0%	2746
4g	light	3832.3	3619.6	0%	162	64	457	1.1%	10800
4g	saturated	4832.8	4702.3	0%	127	43	430	0.9%	10800
5a	light	1262.6	883.7	0%	60	19	101	0%	679
5a	saturated	3457.5	2995.9	0%	47	19	44	0%	356
6g	light	1918.8	456.5	0%	296	284	165	0%	4001
6g	saturated	2226.8	1496.3	0%	273	313	153	13%	10800
9g	light	2043.9	1401.2	2%	394	7214	–	–	–
9g	saturated	4327.5	3344.4	2%	203	7204	–	–	–

[1] Computer Science Lab. of Paris 6 (LIP6), University Pierre-Marie Curie, Paris 6
4 Place Jussieu, 75252 Paris, France. *michel.minoux, hung.nguyen@lip6.fr*

[2] Phoenix-ISI, 2 rue du Centre, 93885 Noisy le Grand Cedex, France.
minh.tuan.ngo@phoenix-isi.fr