

On Nonlinear Dispersive Equations in Periodic Structures: Semiclassical Limits and Numerical Schemes

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Abstract: We discuss (nonlinear) dispersive equations, such as the Schrödinger equation, the Gross-Pitaevskii equation modeling Bose-Einstein condensation, the Maxwell-Dirac system and semilinear wave equations. Semiclassical limits are analysed using WKB and Wigner techniques, in particular for periodic structures, and connections to classical homogenisation problems for Hamilton-Jacobi equations and hyperbolic conservation laws are established. We present a new numerical technique for such PDE problems, based on Bloch decomposition, and show applications in semiconductor modelling, Bose-Einstein condensation and Anderson localisation for random wave equations.

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