Economic Applications of Infinite Horizon Optimal Control Problems. Sufficient Optimality Conditions.

V. Lykina[1] **and S. Pickenhain**[2]

Abstract: In the problems of the economic growth theory it is common to consider the infinite planning time interval. For this reason the general optimal control problem we deal with is the following. Minimize the objective

$$J(x,u) = \int_0^\infty r(t, x(t), u(t))\, \widetilde{\nu}(t) dt$$

subject to $x \in W_p^{1,n}(\mathrm{I\!R}^+, \nu)$, $u \in L_p^m(\mathrm{I\!R}^+, \nu)$, satisfying almost everywhere on $\mathrm{I\!R}^+$ *the state equation*

$$\dot{x}(t) = f(t, x(t), u(t))\,,$$

control constraints

$$u(t) \in U, \quad U \in Comp\,(R^m) \setminus \{\,\emptyset\,\},$$

initial condition

$$x(0) = x_0$$

as well as *pure state constraints*

$$h_l(t, x(t)) \leq 0 \text{ for all } t \in (0, \infty)\,, \; l=1,\ldots,w\,.$$

Integral in the objective function is understood in Lebesgue sense and we follow the weighted Sobolev space approach to establish our results. Two weight functions ν and $\widetilde{\nu}$ are involved into the problem setting, what give us larger state and control spaces as well as the correct correspondence to the adjoint state functions. For this class of control problems we formulate second order sufficient conditions for uniform strong local optimality. It means that the neighbourhood, in which a certain pair (x^*, u^*) is a strong local minimizer, does not depend on time, i.e. the neighbourhood has the minimal breadth. An economical example illustrating the applicability of the proved theorem is presented.

[1,2] Department of Optimization, Institute of Mathematics
Brandenburg Technical University of Cottbus
P.O.B. 10 13 44, 03013 Cottbus, Germany
lykina@math.tu-cottbus.de, sabine@math.tu-cottbus.de