Mathematical Problems of Very Large Networks

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Abstract: Suppose that we have a huge graph (we don't even know its size), and the only way to obtain information about it is to draw a sample of the node set of bounded size. What properties of the graph can be deduced from this sample? What should we mean by an answer to an algorithmic question like finding the connected components, if we cannot even list all nodes? What does it mean if two graphs are "close" in the sense that they cannot be distinguished by such tests? How to model such huge graphs, and how to approximate them by smaller ones?

The graph property testing model was first introduced by Goldreich, Goldwasser and Ron (but related questions were considered before). In the context of dense graphs, a very general result is due to Alon and Shapira, who proved that every hereditary graph property is testable.

In this language, the Regularity Lemma of Szemerédi states that every graph can be approximated by a weighted graph with k nodes so that the error tends uniformly to 0 as $n \to \infty$. In the other direction, the theory of graph limits yields an approximation by a 2-variable measurable function. which allows us to use tools from analysis. This analytic version allows for simpler formulation of many graph theoretic problems, and leads to various characterizations of testable properties.

We survey these results, along with analogous results for graphs with bounded degree.

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