Real-Time Generation Algorithms for Linear-Recursive Sequences on Cellular Automata

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Abstract: Cellular automata (CA) are considered to be a non-linear model of complex systems, computability theory, mathematics and theoretical biology in which an infinite one-dimensional array of finite state machines (cells) updates itself in a synchronous manner according to a uniform local rule. It is studied in many fields such as complex systems. In this paper, We study a **sequence generation problem** on the CA. Let M be a CA and $\{\mathrm{t}_n | n = 1, 2, 3, ...\}$ be an infinite monotonically increasing positive integer sequence defined on natural numbers, such that $\mathrm{t}_n \geq n$ for any $n \geq 1$. We say that M generates a sequence $\{\mathrm{t}_n | n = 1, 2, 3, ...\}$ in k *linear-time* if and only if the leftmost cell of M falls into a special state at time $t = k \cdot \mathrm{t}_n$, where k is a positive integer. We call M a *real-time* generator when $k = 1$.

Arisawa [1971], Fischer [1965], Korec [1998] studied non-regular sequences on CA. Korec [1998] showed that prime sequence can be generated in real-time by a CA with 9 states. Kamikawa and Umeo [2008] showed that sequence $\{2^n \,|\, n = 1, 2, 3, \ldots\}$ can be generated in real-time by a CA with 3 states and sequence $\{n^2 \,|\, n = 1, 2, 3, \ldots\}$ can be generated in real-time by a CA with 4 states, and showed sequence generation power of CA with 2 internal states. In this article, we propose the generation algorithm of linear-recursive sequences, such as Fibonacci, tribonacci, tetranacci and Pell sequences. Let m be any natural number, such that $m \geq 1$. Let k be natural number given, such that $k \geq 1$, $k < m$. Let b_1, b_2, ... , b_k, c_1, c_2, ... , c_k be natural number given, such that $b_1, b_2, \cdots, b_k \geq 1$, $c_1 < c_2 < \cdots < c_k$. Let a_m be kth order linear-recursive sequences, such that $a_m = b_1 \cdot a_{m-1} + b_2 \cdot a_{m-2} + \cdots + b_k \cdot a_{m-k}$, $a_1 = c_1$, $a_2 = c_2$, ..., $a_k = c_k$. We show generation algorithm of sequence a_m on CA. We also study the number of internal states of generation algorithm. We show that an internal state of generation algorithm depends on b_1, b_2, ... , b_k, c_1, c_2, ... , c_k and k. For example, we consider the case of $k = 1$. It is approved that $a_m = b_1 \cdot a_{m-1}$, $a_1 = c_1$. However, it is limited to $b_1 \geq 2$. Because all terms take c_1 for $b_1 = 1$, and a_m is not an infinite monotonically increasing positive integer sequence. When b_1 and c_1 are the even numbers and b_1 is not 2, the number of internal states of generation algorithm of sequence a_m is $4 \cdot b_1 + c_1 - 3$. In this case, sequence a_m can be generate in real-time on CA with $4 \cdot b_1 + c_1 - 3$ internal state.

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