Age Dynamics of Learned Societies and Other Fixed-Sized Populations

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Abstract: In this paper the dynamics of age-structured populations with fixed size is studied. In a hierarchical organization whose total membership size remains constant the annual intake is strictly determined by the number of deaths, quits and a statutory retirement age. There is a fundamental dilemma of two conflicting goals of a constant-sized age-structured population, e. g. an Academy of Sciences: to keep a young age-structure while to guarantee a high recruitment rate.

In this paper we first present a reconstruction of the population of the Austrian Academy of Sciences from 1847 to 2005. Based on alternative scenarios of the age distribution of incoming members we project the population of the Austrian Academy forward in time and study the sensitivity of the total number of members, their age distribution and the number of recruits for those alternative scenarios. Then, we introduce an age-structured optimal control model to determine the optimal trade-off between the rate of replacement and the mean age of a constant-sized population whose dynamics is modelled by the McKendrick partial differential equation. A variant of Pontryagin's maximum principle is derived and used to determine the optimal recruitment distribution in the stationary case. It turns out that due to a U-shaped age-specific shadow price of a member it is optimal to elect either young or old aged new members. We discuss some interesting policy implications of the obtained optimal recruitment policy (scientific excellence and life long achievements).

Furthermore, we study two important issues, namely personal management and migration. In the 1st area a two-level age-and duration-specific optimal control model is an appropriate methodological framework. In particular, we aim to study the dynamics of corresponding members since they constitute the pool from which the predominant number of full members is elected. Questions on the promotion of members can be dealt with in a two-stage model framework. An overly long waiting period or a rather small chance for corresponding members to ever become elected as full members may create frustration and a bad reputation of the academy. Although the optimisation problem for the academy may be decomposed into a cascade of two problems, it is harder to study. Using a multi-compartment model approach we pose the question how to optimally recruit corresponding members and how to promote them to full member status in an efficient way.

Regarding the 2nd topic, we explore the relationships between the age distribution of arriving immigrants and the long- and short-run age distribution of national populations. The relevant methodological framework is the single-level age-structured control model with the age-structured recruitment policy acting as a control variable. An increase in the age of immigrants leads to a lower percentage of young and a higher fraction of old persons. This suggests the existence of an optimal age for minimising the overall dependency ratio.

To summarise the common link of our approach, we analyse the development of age structured fixed-sized populations whose dynamics are governed by age-dependent inflows. Both the transient behaviour as well as the long-run steady state are of interest. In this context our aim is to determine the age-specific inflow rates to guarantee a given target, e.g., a young age structure. The proposed approach can be applied to the dynamics and control of learned societies, universities, and below replacement populations.

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