The Solution of Really Large Sparse Linear Equations from Three-dimensional Modelling

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Abstract: It is often assumed that direct methods of solution cannot be used to solve large systems from three-dimensional modelling. We first show that this conventional wisdom is not entirely well founded by showing that linear systems in millions of unknowns can be solved by a sparse direct method.

It is true, however, that this marks the normal current limit on the size of problems that direct methods can handle and that many modellers want to solve problems of far higher dimension. Even there, direct methods can play an important role; for example, as a coarse grid solver in multigrid or to solve subproblems in a domain decomposition method. We show examples of these approaches and indicate how they can be used to solve even larger problems. These are examples of hybrid methods which combine the raw power and accuracy of a direct method with an iterative method which normally requires much less storage. We briefly discuss this generic class of solution schemes and examine hybrid methods that use a fast but potentially inaccurate sparse factorization.

Researchers in the Parallel Algorithms Group at CERFACS in Toulouse have recently solved three-dimensional Helmholtz problems in seismic modelling with over 65 billion unknowns. We briefly discuss how this has been done.

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