A Second-Order Positivity Preserving Central-Upwind Scheme for Chemotaxis and Haptotaxis Models

A. Chertock[1] and A. Kurganov[2]

Abstract: In this talk, I will present a new finite-volume method for a class of chemotaxis models and for a closely related haptotaxis model. In its simplest form, the chemotaxis model is described by a system of nonlinear PDEs: a convection-diffusion equation for the cell density coupled with a reaction-diffusion equation for the chemoattractant concentration. A common property of all existing chemotaxis systems is their ability to model a concentration phenomenon that mathematically results in solutions rapidly growing in small neighborhoods of concentration points/curves. The solutions may blow up or may exhibit a very singular, spiky behavior. In either case, capturing such singular solutions numerically is a challenging problem.

The first step in the derivation of the new method is made by adding an equation for the chemoattractant concentration gradient to the original system. We then show that the convective part of the resulting system is typically of a mixed hyperbolic-elliptic type and therefore straightforward numerical methods for the studied system may be unstable. We design a new method, which is based on the application of the second-order central-upwind scheme, originally developed for hyperbolic systems of conservation laws, to the extended system of PDEs. We prove that the proposed central-upwind scheme is positivity preserving, which is a very important stability property of the method.

We apply the new method to a number of two-dimensional problems including the most commonly used Keller-Segel chemotaxis model and its modern extensions as well as to a haptotaxis system modeling tumor invasion into surrounding healthy tissue. Our numerical results demonstrate high accuracy, stability, and robustness of the proposed scheme.

[1] Department of Mathematics, North Carolina State University
Cambus Box 8205/HA 243, Raleigh, NC 27695, USA
chertock@math.ncsu.edu

[2] Mathematics Department, Tulane University
6823 St. Charles Avenue, New Orleans, LA 70118, USA
kurganov@math.tulane.edu