Modeling and Optimization in Cryopreservation Using Phase Field Models

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Abstract: Many biotechnologies involve freezing of small tissue samples in such a manner that the cells preserve their biological activity after subsequent thawing. Usually, tissue samples are being frozen in plastic ampoules with appropriate liquids.

Ice formation in the liquid surrounding the tissue sample is the most critical stage of cryopreservation because many injuring effects occur here. One of them is supercooling caused by the recrystallization (conversion of ice nuclei into dendritic structures). Another unpleasant effect is the release of the latent heat that impedes cooling so that the temperature remains constant for a while and then drops suddenly.

Nowadays, phase field techniques for modelling of freezing processes become very popular. They have been introduced by G. Caginalp and studied by many scientists (see e.g. K.-H. Hoffmann and Jiang Lishang. Numer. Funct. Anal. Optimiz., 13, 1&2, 1992, 11–27).

Let Ω be the interior of the ampoule (assume first the absence of the tissue sample), Γ the boundary of Ω. The model equations look as follows:

$$\begin{aligned}
&u_t + \tfrac{\ell}{2}\phi_t - K\Delta u = 0, && x \in \Omega,\\
&\tau\phi_t - \xi^2\Delta\phi - \tfrac{1}{2}(\phi - \phi^3) - 2u = 0, && x \in \Omega,\\
-&K\frac{\partial u}{\partial n} = h(u - u_e(t) - g), \quad \frac{\partial \phi}{\partial n} = 0, && x \in \Gamma,\\
&u|_{t=0} = u_0, \quad \phi|_{t=0} = \phi_0.
\end{aligned}$$

Here, u is the scaled distribution of the temperature; ϕ the phase function: $\phi = 1$ for the solid state and $\phi = -1$ for the liquid state; ℓ the scaled latent heat; K the scaled heat conductivity coefficient; h the scaled overall heat conductivity; g the boundary control (add-on to the nominal cooling protocol $u_e(t) = u_0 + \theta_0 t$, where $\theta_0 < 0$ is a given slope).

We prove the existence and uniqueness of solutions in the case of L^2-regularity of the initial data and under the assumption that Γ is of the class $C^{0,1}$, i.e. Lipschitz continuous. Such an assumption covers various technical designs of ampoules and permits the direct extension of the model to the case where a solid object with irregular boundary (tissue) is immersed into the fluid. Mathematical correctness of control techniques based on adjoint equations is discussed in the case of the above-formulated relaxed requirements. Results of optimization are demonstrated.

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