Robust Parameter Estimation based on Huber Estimator in Systems of Differential Equations

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Minisymposium: Optimization and Engineering

Abstract: The problem of identification of unknown parameters in dynamic models is among the most important tasks in mathematical modeling of dynamic processes. The identification problem can be described as follows. Let the dynamics of the model be described by a system of ordinary differential equations $\dot{x} = f(t, x, p)$, where the right-hand side f depends on an unknown vector of parameters p. It is assumed that there is a possibility to measure a signal η of an output device that writes at given time points t_j, $j = 1, .., k$, the output signal $q(t_j, x(t_j), p)$, of the dynamic system with some error $\varepsilon(t_j) : \eta(t_j) = q(t_j, x(t_j), p)+ \varepsilon(t_j)$, $j = 1, ..., k$. According to the common approach, in order to determine the unknown parameters the optimization problem is solved in which the special functional is minimized under constraints that describe the specifics of the model. A suitable norm of the measurement error may be used as the functional in the optimization problem. If the errors are independent, normally distributed with zero mean and known variances σ_j^2, minimizing a weighted least squares function

$$\min_p \sum_j (\eta(t_j) - q(t_j, x(t_j), p))^2 / \sigma_j^2$$

yields a *maximum likelihood estimate*. In practical applications, however, it often appears that the data contains outliers. Thus, a reliable parameter estimation procedure is necessary that delivers parameter estimates less sensitive (robust) to errors in measurements. One of the robust parameter estimators is based on l_1 estimation:

$$\min_p \sum_j |\eta(t_j) - q(t_j, x(t_j), p)| / |w_j|$$

The other possibility is to use the so-called Huber estimator that combines l_1 and l_2 estimators:

$$\min_p \sum_j \rho_H((\eta(t_j) - q(t_j, x(t_j), p))/w_j)$$

where $\rho_H(z) = z^2$, if $|z| \leq \gamma$, $\rho_H(z) = \gamma|z| - \gamma^2$, if $|z| > \gamma$.

The paper focuses on the method for robust parameter estimation for dynamic systems which combines the boundary value problem approach and a special method of solving Huber approximation problems. The method is successfully applied to several real-life problems.

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