Branching Techniques and Valid Linear Inequalities for Mixed-Integer Nonconvex Programming

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Abstract: Mixed-Integer Nonlinear Programming (MINLP) problems arise in fields such as Chemical Engineering, Portfolio Optimization, and Bioengineering, only to name a few. Solving nonconvex MINLPs requires to find the minimum of a nonconvex function subject to nonconvex constraints and to the integrality of a subset of variables. One notable difficulty is that even relaxing the integrality of all variables yields a nonconvex NLP, which is only solved to optimality by implicit enumeration of all local minima.

Branch-and-bound is a method of choice for nonconvex MINLP problems. We have implemented *Couenne*, an Open Source branch-and-bound algorithm available within the COIN-OR framework (`www.coin-or.org`). *Couenne* relies on a linearization of the problem, generated by applying well-known techniques of reformulation of the original problems.

A crucial component of the branch-and-bound is a set of branching rules for nonlinear terms and integer variables. *Couenne* extends *reliability branching*, a technique that proved successful for Mixed-Integer Linear Programming, to branching on continuous variables; this requires a more careful handling of pseudocosts. Also, we present a generalization of *disjunctive cuts* to MINLP. Disjunctions are generated from branching rules on continuous and integer variables and by applying reduction techniques to the solution set. Some computational results on MINLP instances from well-known libraries `minlplib` and `globallib` are discussed to show the impact of both techniques on the solution time.

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