Free Boundary Problems in Thin Film Hydrodynamic Lubrication Problems

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Abstract: Mathematical models in hydrodynamic lubrication are mainly governed by the so called Reynolds equation which gives the pressure field and then the velocity fields in a situation in which the gap in the direction normal to the flow is much smaller than the other dimensions of the device. For example, this description can be used for pistonring, roller bearing, slider bearing, magnetic tape. Most often, Reynolds equation is an elliptic linear equation so that its solution is easily computed without mathematical difficulty. However due the boundary conditions and geometrical shapes involved, negative pressure (pressure lower than the saturation pressure of the fluid) are often obtained which are far to be physically correct. The reason of this discrepancy is that Reynolds equation is a mathematically correct approximation of the Stokes system, that is to say a description valid for a homogeneous fluid. However, in most of the situation in which lubrication is concerned with, this is not the case as the observation of the device shows that in some areas some bubble or mushy region appears and a biphasic description is needed. The fact that in this mushy region, pressure is near constant (zero) and always greater than zero in the other homogeneous area leads to the introduction of various free boundary models always based on the Reynolds equation: This include first kind variational inequality, parabolic hyperbolic pressure saturation system. More recently, another system has been proposed based upon a mathematically rigorous approach of a thin film biphasic Stokes system. This leads to a new model, in which a non linear hyperbolic equation allows us to describe the interface between two-fluid of different viscosities while a modified Reynolds equation gives the pressure. Numerical computation of the solutions of the various models will be given.

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