Infinite Horizon Optimal Control Problems – Lebesque and Riemann Improper Integrals

S. Pickenhain[1] **and V. Lykina**[2]

Abstract: In this paper we consider infinite horizon optimal control problems of the following type:

$$J(x,u) = \int_0^{\infty} f_0(t,x(t),u(t))\,dt \;\longrightarrow\; \text{Min}\,! \tag{1}$$

subject to $x \in W^{1,n}_{p,\nu}(0,\infty)$, $u \in L^r_{p,\nu}(0,\infty)$, satisfying a. e. on $(0,\infty)$

state equations

$$x'(t) = g(t,x(t),u(t))\,,$$

control restrictions

$$u(t) \in U, \quad U \in Comp\,(R^r) \setminus \{\,\emptyset\,\},$$

initial conditions

$$x(0) = x^0.$$

Different criteria of optimality are known for such problems, e. g. the overtaking criterion of v. Weizsäcker (1965), the catching up criterion of Gale (1967) and the sporadically catching up criterion of Galkin (1974). Corresponding to these criteria we prove sufficient conditions for local optimality. Here local optimality is understood in the sense weighted Sobolev spaces $W^{1,n}_{p,\nu}(0,\infty)$, i.e.

$$\|x - x^*\|^p_{W^{1,n}_{p,\nu}(0,\infty)} := \int\limits_0^{\infty} \{\|(x(t) - x^*(t)\| + \|(x'(t) - {x^*}'(t)\|\}^p \nu(t)dt < \epsilon,\, \epsilon > 0$$

with a density function ν, with $0 < \nu(t) < \infty, \quad \text{a.e. on } (0,\infty)$ and $\int\limits_0^{\infty} \nu(t)dt < \infty$.

Some aspects concerning the integral in (1) are essential. We allow both Lebesgue and improper Riemann integrals to appear in integrand of the objective. It can happen, that the integral in (1) as Lebesgue integral does not exist and at the same time the Riemann integral is convergent. We give applications where the improper Riemann integral represents the adequate formulation of the problem and discuss analytic properties of the objective functional in Riemann and Lebesgue sense.

[1,2] Department of Optimization, Institute of Mathematics
Brandenburg Technical University of Cottbus
Karl - Marx - Str. 17, 03046 Cottbus, Germany
lykina@math.tu-cottbus.de, sabine@math.tu-cottbus.de